



# **Bi-level Decision Making with Fuzzy Sets and Particle Swarm Optimisation**

**A thesis submitted for the degree of  
Doctor of Philosophy**

By

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## **CERTIFICATE OF AUTHORSHIP/ORIGINALITY**

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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## Abstract

Bi-level programming techniques are developed for decentralized decision problems with decision makers located in a two-level decision making system; the upper decision maker is termed the leader while the lower is the follower. Both the leader and the follower try to optimise their own objective functions and the corresponding decisions do not control but do affect those of the other level.

This research aims at solving bi-level decision problems with five extensions, i.e. multiple leaders/followers/objectives, fuzzy coefficients and goals. By using particle swarm optimisation and/or cut set and/or goal programming and/or Nash equilibrium concept, related mathematical models and corresponding algorithms are developed to solve fuzzy linear bi-level decision problems, fuzzy linear multi-objective bi-level decision problems, fuzzy linear multi-follower multi-objective bi-level decision problems, fuzzy linear goal bi-level decision problems, multi-leader one-follower bi-level decision problems, one-leader multi-follower bi-level decision problems, and multi-leader multi-follower bi-level decision problems. A fuzzy bi-level decision support system is then developed which implements all the algorithms to support bi-level decision making with different features. Finally, by using these bi-level models and algorithms, we explore possible applications in the fields of railway train set organisation, railway wagon flow management, strategic bidding in the electricity market, and supply chains to solve real world bi-level decision problems. The results of experiments show that the models and algorithms are effective for solving real world bi-level decision problems.

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